The elastic and the plastic models predict similar values of contact conductance for this pair, and the elastic model predicts the first loading very well, especially at high contact pressures. However, the appearance of the hysteresis loop clearly shows that the deformation is plastic during the first loading. If one simply compares first loading data with the elastic model, the good agreement could suggest that the deformation of the asperities is elastic, which is not true. On the other hand, when a complete loading/unloading cycle is measured, it is easy to verify that the deformation mode of the contacting asperities is plastic in this case. The data points for first unloading and second loading/unloading cycle lie well above the models, as expected, due to the plastic deformation experienced by the asperities during first loading.

The plastic model presents the same behavior when compared with both test pairs: For the first loading, it underpredicts the experimental data at light loads, but as the pressure increases, the theoretical prediction gets closer to the measured values. This observation is in agreement with the experimental data compiled by Sridhar and Yovanovich.[1] Because this phenomenon has been consistently detected by different researchers employing different setups, it does not seem to be a weakness of the experimental program adopted here. The present authors believe that this is a weakness of the theoretical models. The theoretical models assume a Gaussian asperity height distribution, but the authors believe that the highest asperities of the real surfaces are truncated. At light contact pressures, only the higher asperities come into contact, and the truncation of the highest asperities makes the mean separation between the contacting surfaces smaller than predicted by the Gaussian model. Because the actual separation is smaller than predicted, the actual thermal contact conductance is higher than predicted by the Gaussian model, especially at light contact pressures. As the contact pressure increases, more and more asperities come into contact, and the effect of the few truncated asperities becomes negligible. A new thermal contact conductance model that takes the effect of the truncation of the contacting asperities into account is needed.

Conclusions

The appearance of the hysteresis loop indicated that the contact between bead-blasted/lapped SS 304 is plastic during the first loading/unloading cycle for both roughness levels tested, 0.72 and 1.31 μm. The plastic model of Cooper et al.[2] predicted first loading data points very well for high contact pressures. For light contact pressures, the model underpredicts the experiments. Other researchers employing different experimental setups have systematically noticed this unexpected behavior, indicating that this is a weakness of the theoretical models. The present authors believe that the models underpredict the experiments at light loads due to the truncation of the highest asperities; the highest asperities are shorter than predicted by the models. A new model is needed for the light contact pressure range.

Acknowledgments

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Cooling Fin Design

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Nomenclature

A	=	area of the fin cross section, m^2
b	=	q/kT_a, m^{-1}
h	=	heat transfer coefficient, W/Km^2
L	=	fin length, m
k	=	thermal conductivity, W/Km
m	=	fin parameter, m^{-1}
P	=	perimeter of the fin cross section, m
q	=	heat flux at the fin root, W/m^2
T	=	fin temperature, K
T_a	=	ambient temperature, K
T_L	=	tip temperature, K
T_0	=	temperature at the fin root, K
x	=	dimensionless position along the fin
z	=	position along the fin, m
θ	=	dimensionless temperature, $(T - T_a)/T_a$
θ_L	=	dimensionless temperature at the tip, $(T_L - T_a)/T_a$
θ_0	=	dimensionless temperature at the fin root, $(T_0 - T_a)/T_a$

Introduction

RECENT advances in high-performance designs for everything from electronic components at the submicron scale to equipment used in aircraft and space vehicles have increased the need for enhanced heat transfer devices. The design and analysis of fin structures for extended surface heat transfer are at the forefront of this technology. Although the basics of the heat conduction process in straight fins are extensively discussed in standard heat transfer textbooks,[1] the design of specific applications often requires a more detailed analysis leading to a broad range of surface types and operating conditions.[2]

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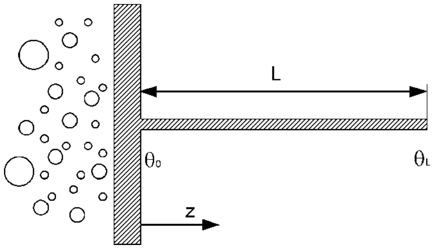

Fig. 1 Schematic representation of a fin cross section; the boundary condition at the fin root is imposed by a fluid undergoing a gas–liquid phase transition.

The solution of any heat conduction problem in a cooling fin is given by the differential equation[1]

$$\frac{\mathrm{d}^2(T-T_a)}{\mathrm{d}z^2}=\frac{hP}{kA}(T-T_a) \tag{1}$$

where A and P account for the geometry of the fin cross section. The solution of this equation represents the temperature distribution along the fin for selected initial and boundary conditions.

An interesting one-dimensional, steady-state problem of heat conduction arises in designing a cooling fin with fixed temperature boundary conditions at both ends and specified heat flux at the root. This set of conditions can be realized if the fin is connected to a wall where a phase change occurs, as is schematically depicted in Fig. 1. Whereas from the purely mathematical standpoint the direction of the heat flux is not important, in practical applications the flux is always directed from the fluid to the fin; that is, the fin is used to dissipate thermal energy transported by the fluid. This happens when a vapor flow undergoes homogeneous condensation or when a liquid flow starts boiling due to internal heat sources (for instance, chemical or nuclear reactions). The latter circumstance is notably important, because it is a typical consequence of an accident, and the cooling fin must act as a passive safety device, dissipating the unwanted excess heat.

In such cases, the heat flux at the fin root may change considerably while the wall temperature is almost constant.[3] Thus, the second-order heat equation must satisfy three boundary conditions so that an additional degree of freedom must be introduced in order to achieve a well-posed problem. For instance, a natural choice is to consider the fin length as a variable. In this Note the analytical solution of this problem is discussed as a function of the heat flux. A numerical example is also presented. The results indicate that, depending on the value of the heat flux at the fin root, the problem of heat conduction in a fin of variable length may have no solutions, one solution, or two solutions.

Analysis

The steady-state, dimensionless conductive problem for the fin is as follows:

$$\ddot{\theta}(x)=(Lm)^2\theta(x),\qquad \dot{\theta}(0)=-Lb,\qquad \theta(0)=(T_0-T_a)/T_a$$
$$\theta(1)=(T_L-T_a)/T_a,\qquad 0\le x\le 1 \tag{2}$$

where $x=z/L$, $m^2=hP/kA$, and $b=q/kT_a$.

The solution that satisfies the differential equation and the boundary conditions at the fin root ($x=0$) is

$$\theta(x)=(\theta_0-b/m)\cosh(Lmx)+(b/m)e^{-Lmx} \tag{3}$$

The boundary condition at the tip allows one to derive a quadratic in e^{mL},

$$(\theta_0-b/m)e^{2mL}-2\theta_L e^{mL}+(\theta_0+b/m)=0 \tag{4}$$

that can be used to calculate the fin length. The solutions of Eq. (4) must be real and positive to represent a physical length. In particular, if $b=m\theta_0$ the one positive solution is

$$L=(1/m)\ln(\theta_0/\theta_L) \tag{5}$$

whereas if $b\neq m\theta_0$ the solution is

$$e^{mL}=\frac{\theta_L\pm\sqrt{\theta_L^2-\theta_0^2+(b/m)^2}}{\theta_0-(b/m)} \tag{6}$$

so that no real solutions exist if $b<m\sqrt{(\theta_0^2-\theta_L^2)}$. Positive solutions for the fin length are obtained if the right-hand side of Eq. (6) exceeds unity, requiring

$$\frac{\theta_L-\theta_0+(b/m)\pm\sqrt{\theta_L^2-\theta_0^2+(b/m)^2}}{\theta_0-(b/m)}>0 \tag{7}$$

Suppose first that $b>m\theta_0$ holds. The largest root corresponds to the plus sign, and Eq. (7) is physically realizable if

$$\sqrt{\theta_L^2-\theta_0^2+(b/m)^2}<\theta_0-\theta_L-b/m \tag{8}$$

Squaring both sides leads to the contradiction $b<m\theta_0$.

On the other hand, the smallest solution gives

$$\sqrt{\theta_L^2-\theta_0^2+(b/m)^2}>\theta_L-\theta_0+b/m \tag{9}$$

which is always true for $b>m\theta_0$; thus, there is one positive solution for the fin length if $b>m\theta_0$.

Suppose next that $b<m\theta_0$. Equation (7) with the plus sign gives the condition

$$\sqrt{\theta_L^2-\theta_0^2+(b/m)^2}>\theta_0-\theta_L-b/m \tag{10}$$

which is always true for $b<m\theta_0$, whereas the minus sign gives the condition

$$\sqrt{\theta_L^2-\theta_0^2+(b/m)^2}<\theta_L-\theta_0+b/m \tag{11}$$

which for $b<m\theta_0$ is always true as well. Thus, in this case there are two solutions. In conclusion, we have no solution for $b<m\sqrt{(\theta_0^2-\theta_L^2)}$, one solution for $b\ge m\theta_0$, and two solutions for $m\sqrt{(\theta_0^2-\theta_L^2)}<b<m\theta_0$.

Example

Consider a cylindrical fin attached to a wall where a water flow undergoing a liquid-to-gas phase change at atmospheric pressure occurs and suppose the phase transition is due to some undesired internal heat generation that must be removed by the fin. The temperature at the fin root is 373 K due to the phase change. Suppose that the temperature of the tip must be 323 K, and the temperature of the environment surrounding the fin is 293 K, so that $\theta_0=0.273$ and $\theta_L=0.102$. For simplicity, let $m=1$.

Values of the fin length as a function of the parameter b are shown in Fig. 2. For $b<0.2532\ \mathrm{m}^{-1}$ there are complex solutions, whereas for $0.2532\ \mathrm{m}^{-1}<b<0.273\ \mathrm{m}^{-1}$ there are two real solutions, and for $b>0.273\mathrm{m}^{-1}$ one real solution; the largest solution becomes asymptotically infinite for $b=0.273\ \mathrm{m}^{-1}$. The value of the heat flux in real applications, of course, must be smaller than the critical heat flux value, at which wall burnout would occur.[4] Although an infinite length of the fin is of no use for practical applications, the existence of two finite values of the fin length in a given range of heat fluxes provides an additional design parameter that could be exploited under particular circumstances. The same situation is depicted from a different standpoint in Fig. 3, which shows the values of the dimensionless temperature of the tip with respect to the fin length.

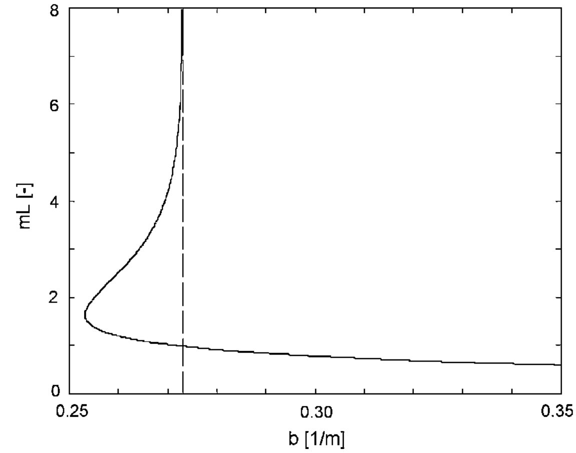

Fig. 2 Values of the total dimensionless fin length mL satisfying all of the boundary conditions, as a function of the parameter b, proportional to the heat flux at the fin root.

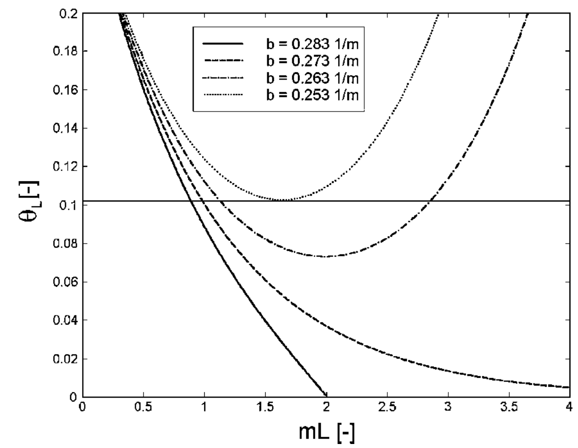

Fig. 3 Admissible values of the dimensionless temperature at the tip as a function of the fin length for different values of the parameter b; the solution of the problem is given by the intersection of the curves with the horizontal line at $\theta_L = 0.102$.

Conclusions

Although the heat conduction problem in cooling fins is usually considered to have one steady-state solution for given boundary conditions, two solutions may exist when nonstandard, yet physically admissible, boundary conditions are applied. In particular, the situation in which the cooling fin has fixed temperature at both ends and specified heat flux at the root is examined. The analysis shows that, depending on the value of the heat flux at the fin root, the problem may have no solutions, one solution, or two solutions for the fin length. The results can find practical applications in the design of passive safety devices for chemical or nuclear plants.

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Integral Equation for the Heat Transfer with the Moving Boundary

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Nomenclature

A	=	heated surface area, m^2
b	=	mean of normal distribution, s
c_p	=	specific heat of solid material
E	=	activation energy, J
F	=	fluence of the laser, J
K	=	preexponent
k	=	thermal conductivity, W m^{-1} K^{-1}
p	=	pressure, Pa
q''	=	local heat flux, W m^{-2}
q''_s	=	surface heat flux (at $x = 0$), W m^{-2}
R	=	gas constant
r	=	reflectivity of the material
T	=	absolute temperature, K
$T_{\max}$	=	maximum temperature during heating, K
T_s	=	surface temperature (at $x = 0$), K
T_0	=	initial temperature (at $t \leq 0$), K
t	=	time, s
u	=	speed of the domain boundary, m s^{-1}
x	=	coordinate normal to material surface, m
α	=	thermal diffusivity, m^2 s^{-1}
θ	=	excess temperature, $= T - T_0$, K
ϑ	=	normalized nondimensional temperature, $= (T_s - T_0)/(T_m - T_0)$
μ, ν	=	given parameters for a particular fuel
ρ	=	density, kg m^{-3}
σ	=	variance of normal distribution, s

Introduction

PROBLEMS of heat transfer in domains with moving boundaries are commonly found in scientific and engineering applications. As typical examples, one can consider the following classes of problems:

1) Stefan-type and related problems. These typically arise when there is a phase change at the boundaries between media with different conducting properties, for example, during melting/solidification of alloys or warming/freezing of water-containing soils.

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